Fault-tolerant Control of Multi-hop Control Networks

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Outline

• Challenges in wireless control networks

• Integrated modeling of dynamical systems and communication protocol

• Fault tolerant stabilizability of wireless control networks

• Optimal control of wireless control networks
Wireless Control Systems

Challenge: close the loop around wireless multi-hop control networks.
Wireless Control Systems

Challenge: close the loop around wireless multi-hop control networks.
HYCON2 ongoing EU NoE

Green Buildings Automation

• The design of a green building is achieved by means of the following ingredients:
  – lighting control exploiting natural light or motion detection sensors
  – high-efficiency light fixtures
  – automated HVAC control
  – high-efficiency heating equipment
WirelessHART MAC layer (scheduling)

- Time is divided in periodic frames, each divided in \( \Pi \) time slots, each of duration \( \Delta \)
- To avoid interference, a periodic scheduling allows each node to transmit data only in a subset of time slots
- Model impact of scheduling on the closed-loop dynamics

![Diagram showing periodic frames and time slots]
WirelessHART MAC layer (scheduling)

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![Diagram showing cycle, slots, and node connectivity](image-url)
WirelessHART MAC layer (scheduling)

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Redundancy in the data routing (flooding) and network coding

- Makes the system robust w.r.t. nodes/links failures
- Enables detection of malicious attacks to the wireless nodes/links
- Model impact of routing on the closed-loop dynamics
• Extends the classical controller-plant feedback control scheme
• Control signals sent to the plant via a controllability network
• Measured data sent to the controller via an observability network

[D’Innocenzo, Di Benedetto, Serra, IEEE-TAC-13]
Input/Output MCN Model

\[ \mathcal{P} = (A, B, C) \]

Syntax:
- LTI SISO plant
Input/Output MCN Model

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- LTI SISO plant
- Weight function $W$ determines data processing through network - network coding

$G_R = (V_R, E_R, W_R)$
$W_R: E_R \rightarrow \mathbb{R}$

$\mathcal{P} = (A, B, C)$

$G_O = (V_O, E_O, W_O)$
$W_O: E_O \rightarrow \mathbb{R}$
Input/output MCN Model

**Syntax:**
- LTI SISO plant
- Weight function $W$ determines data processing through network
- Communication scheduling $\eta$ assigns transmission of nodes
Input/Output MCN Model

Syntax:
- LTI SISO plant
- Weight function $W$ determines data processing through network - network coding
- Communication scheduling $\eta$ assigns transmission of nodes
- Model at time scale of frames instead of time-slots (no switching behavior)

Mathematical Expressions:
- $G_R = (V_R, E_R, W_R)$
- $W_R: E_R \rightarrow \mathbb{R}$
- $\eta_R: \{1, ..., \Pi\} \rightarrow 2^{E_R}$
- $P = (A, B, C)$
- $G_O = (V_O, E_O, W_O)$
- $W_O: E_O \rightarrow \mathbb{R}$
- $\eta_O: \{1, ..., \Pi\} \rightarrow 2^{E_O}$
- $T = \Pi \Delta$
MCN interconnected model

Model the semantics of a MCN $N$ by cascade of discrete time SISO LTI systems, with sampling time equal to the scheduling period duration.
Time slot level network dynamics

\[ W_{2,u} = B_1 w_{1,2} + B_3 w_{3,2} \]
Time slot level network dynamics
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\[ W_{2,u} = B_1 w_{1,2} + B_1 w_{3,2} \]

\[ t + 1 \]
Time slot level network dynamics

\[ B_1 w_{1,2} + B_1 w_{3,2} \]

\[ t+2 \]
Time slot level network dynamics

\[ B_1 w_{1,2} + B_1 w_{3,2} \]

12 bits

18 bits

14 bits

\[ B_1 (w_{1,2} + w_{3,2}) B_1 \]
Frame level network dynamics

Frame 1

Direct Scheduling

\[ u(kT) = \tilde{u}((k-1)T) \]

\[ G_R(z) = \frac{1}{z} \]
Frame level network dynamics

Reverse Scheduling

Frame 1
| $v_2, v_3$ | $v_1, v_2$ |

Frame 2
| $v_2, v_3$ | $v_1, v_2$ |

$k = 2$
$h = 2$

$u(kT) = \tilde{u}((k-2)T)$

$G_R(z) = \frac{1}{z^2}$

we need 2 frame periods to convey the data from $v_1$ to $v_3$
Frame level network transfer function

**Proposition:** Given a MCN N, the controllability network $G_R$ can be modeled as a discrete time SISO LTI system with sampling time equal to the scheduling period duration $\Pi\Delta$, and characterized by the following transfer function:

$$G_R(z) = \sum_{d=1}^{D_R} \frac{\gamma_R(d)}{z^d}, \quad \gamma_R(d) = \sum_{\rho \in \chi_R(d)} W_R(\rho)$$

where $\chi_R(d)$ is the set of paths of $G_R$ characterized by delay $d$, $D_R \in \mathbb{N}$ is the maximum delay introduced by the paths of $G_R$, $W_R(\rho)$ is the product of weights of all links that generate path $\rho$ of $G_R$, and $\forall d \in \{1, \ldots, D_R - 1\}, \gamma_R(d) \in \mathbb{R}_0$, $\gamma_R(D_R) \in \mathbb{R}$.

$$\rho = e_1, e_2, e_3, e_4$$

$$W_R(\rho) = W_R(e_1)W_R(e_2)W_R(e_3)W_R(e_4)$$
If all paths have the same delay...

• The cascade of systems $G_R$ and $P_T$ always satisfies the controllability condition
Why not designing all delays to be equal?

\[ G_1(z) = \frac{1}{z} \]

\[ \Pi_1 = 3 \quad \Rightarrow \quad \Pi_1 \Delta = 30ms \]
Why not designing all delays to be equal?

\[ G_1(z) = \frac{1}{z} \]
\[ \eta_1 \rightarrow \quad \Pi_1 = 3 \rightarrow \quad \Pi_1 \Delta = 30ms \]

\[ G_2(z) = \frac{0.6z + 0.4}{z^2} \]
\[ \eta_2 \rightarrow \quad \Pi_2 = 2 \rightarrow \quad \Pi_2 \Delta = 20ms \]
Example

\[ \Pi^c = 2 \quad T^c = 20 \text{ ms} \]

\[ \eta^c_{3R} = \langle \{(v_1, v_2), (v_1, v_3), (v_1, v_4), (v_5, v_7), (v_6, v_7)\}, \{(v_2, v_5), (v_2, v_7), (v_3, v_5), (v_3, v_6), (v_3, v_7), (v_4, v_6), (v_4, v_7)\} \rangle. \]

\[ u(kT_c) = \frac{3}{5} \tilde{u}((k - 1)T_c) + \frac{2}{5} \tilde{u}((k - 2)T_c) \]
Example

\[ P_{Tc}(z) = 4.2932 \times 10^{-9} \frac{z + 1.189}{(z + 1)(z + 2)}, \]

\[ G_{TR}(z) = \frac{\frac{3}{5}z + \frac{2}{5}}{z^2} \Rightarrow z = -\frac{2}{3} \]

\[ f_2 = \{(v_3, v_7), (v_5, v_7)\} \Rightarrow G_{TR}^{f_2}(z) = \frac{\frac{2}{5}z + \frac{1}{5}}{z^2} \Rightarrow z_{f_2} = -\frac{1}{2} \]
**Example**

\[ P_{Tc}(z) = 4.2932 \times 10^{-9} \frac{z + 1.189}{(z + 1)(z + 2)}, \]

\[ G_{TR}(z) = \frac{\frac{3}{5}z + \frac{2}{5}}{z^2} \Rightarrow z = -\frac{2}{3} \]

\[ f_3 = \{(v_3, v_7), (v_2, v_7)\} \Rightarrow G_{TR}^{f_3}(z) = \frac{\frac{1}{5}z + \frac{2}{5}}{z^2} \Rightarrow z_{f_3} = -2 \]
Fault tolerant stabilizability of MCN

Assumptions:

- No fault detection algorithms in the network protocol: only use input to and output from the MCN
- Failures are slow with respect to plant time constants
Fault tolerant stabilizability of a MCN

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Problem 1: Guarantee existence of a stabilizing controller for the MCN dynamics $M_f$ associated to any $f \in F$

Problem 2: Design a dynamical system (FDI) able to detect and isolate any $f \in F$
Problem: Design a weight function $W_R$ such that the fault MCN $M_f$ is stabilizable for any $f \in F$.

Theorem: Given a SISO MCN, Problem 1 can be solved if and only if:
1. Plant pair $(A,B)$ is controllable
2. At least one scheduled path connects the controller with each actuator (condition on network topology and on scheduling function $\eta_R$)
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Extension to MIMO systems

[Smarra, D’Innocenzo, Di Bendetto, NecSys-12]
Problem: Translate geometric conditions for Extended Fundamental Problem in Residual Generation [Massoumnia et al. TAC89] to conditions on the network topology

Theorem: Given a SISO MCN Problem 2 can be solved if and only if:
1. No failures occur in the controllability network
2. The observability network topology is a rooted tree

[D'Innocenzo, Di Benedetto, Serra, IEEE-CDC-ECC-11]
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Extension to the MIMO case [Submitted to IEEE-CDC-13]
Consider transient response to a Heaviside Unit Step Function reference

**Problem:** Given a scheduling $\eta_{\mathcal{R}}$, design digital controller $C(z)$ and weight function $W_{\mathcal{R}}$ to minimize the $L_2$-norm of the error signal, with constraints on the control effort, on the overshoot and on the bandwidth exploited in the communication links.

**Theorem:** The above problem is in general NP-hard. Sufficient conditions on network topology, scheduling and routing such that it reduces to a convex constrained optimization problem.

[Smarra, D'Innocenzo, Di Benedetto, IEEE-CDC-12]
**Problem:** Design scheduling $\eta_{\mathcal{R}}$, digital controller $C(z)$ and weight function $W_{\mathcal{R}}$ to minimize the constrained optimization defined in Problem 1.

**Proposition:** The $L_2$-norm induces a total ordering in the finite set of scheduling functions: solve Problem 1 for each scheduling and choose $\eta_{\mathcal{R}}$ associated to the minimal $L_2$-norm. The problem is in general combinatorial.

[Smarra, D'Innocenzo, Di Benedetto, IEEE-CDC-12]
Conclusions

• Unifying framework for modeling dynamical systems and communication protocols

• Co-design of control algorithms and communication protocol configuration to stabilize and satisfy transient response specifications on wireless control networks

• Future work: modeling of physical layer (e.g. packet loss)