Traffic control show case

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Workshop #3 - Control of large-scale distributed and cooperating systems: Recent achievements within the Network of Excellence HYCON2
Outline

Part I: the traffic show case
- Technical details of the show case
- Challenges

Part II: the control algorithms
- Introduction on ramp metering
- Standard MPC
- Event-triggered MPC
- Cluster-based distributed MPC
- Conclusions
Outline

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Collaborations between partners

Active nodes:
- From HYCON2
- External collaborations

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Grenoble Rocade (E/W south ring)

- 10.5 km, 2ways-2lines, serving 90000 veh/day
- Daily Travelling Time variations: from 7-to-50 min
- In Grenoble > 50% NOx-pollution due to traffic

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Sensor deployment

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Technical data

Flow = veh/h
Velocity = km/h
Vehicle length & gap

TECHNOLOGY

130 SENSYS
17 NODES
45 CELLS (model)
4 Full Equipped Junctions
4 In-Ramp Queues

⇢ = On-ramps
⇠ = Off-ramps
Real-time data (17, Sept 17:00)

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Real-time data (17, Sept 17:00)

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Max capacity 900 Vehicles in the E/W direction
- 15 Vehicles/in-ramp on queues
- Total “control” 13% of the occupancy at rush hours
Actuators VSL panels

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Micro simulation is used for:

- Validate concepts and models
- Evaluate forecasting & predictions
- Evaluate & compare control algorithms
Collaborative software remote platform

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Challenges

Models
- Model validation using real-data
- Reduce model complexity

Traffic forecasting
- Demand/velocity prediction
- Dealing with data and model uncertainty

Traffic control
- Event-triggered/Distributed
- Modular
- Suitable for field-implementation
Traffic forecasting (TT-Prediction)

- Velocity-based method (extendable to FCD-sensors)
- Flow-based (counting) estimation
- Robust forecasting
Achievements (highlights)

1. MPC-Numeric (UNIPV, DELFT, US, AQUILA, GENOVA)
   - Distributed cost: reduction of computation time
   - Event triggered & driven: act only when necessary
   - Variable time-horizon: communication delays & data losses

2. Optimization with feedback structure (INRIA)
   - Optimal balancing
   - Distributed optimization via non-cooperative games
   - Information exchange accounts for controllability properties
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Ramp metering is a control action based on the use of traffic lights at the on-ramps in order to regulate the traffic volumes entering the mainstream.

Ramp metering has been in use for some decades.

It has been shown that with ramp metering it is possible to prevent congestion and reduce the travel times of vehicles in the freeway.

Different control approaches have been studied, from simple control laws to very sophisticated control schemes.
Minimization of the **Total Time Spent** by the drivers in the network (both in the mainstream and at the on-ramps) [veh h]

Maximization of the **Total Traveled Distance** by the drivers in the network [veh km]

Tracking of **set-point values**

**Other** objectives (safety, emissions, and so on)
One of the most widely used freeway traffic control approaches is **Model Predictive Control**.

**Advantages:** prediction capability, optimality (suboptimality), compliance with the constraints.

**Drawbacks:** computational load and hence difficulty to be applied in real time.
Our works based on MPC

1. To reduce the computational and communication load of the MPC algorithms for freeway control, we have developed an event-triggered MPC approach.

2. In order to face specific events occurring in the freeway, we have proposed an event-based MPC scheme (see Ferrara et al, SMC 2013).

3. To deal with very large scale freeway systems we have developed a cluster-based distributed MPC scheme.

All the algorithms proposed during the Hycon2 project have been tested in simulation both considering portions of the Grenoble South Ring (Rocade sud) and other European freeways.


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CTM as prediction model

\[ \rho_i(h) = \rho_{i-1}(h) + \phi_i(h) - r_i(h) - s_{i-1}(h) + d_i(h) \]

- \( \rho_i(h) \) is the traffic density of cell \( i \) [veh/km]
- \( l_i(h) \) is the queue length in the on-ramp of cell \( i \) [veh]
- \( \phi_i(h) \) is the mainstream flow entering cell \( i \) from cell \( i-1 \) [veh/h]
- \( r_i(h) \) is the flow entering cell \( i \) from the on-ramp [veh/h]
- \( s_i(h) \) is the flow exiting cell \( i \) through the off-ramp [veh/h]
- \( d_i(h) \) is the on-ramp demand referred to cell \( i \) [veh/h]
CTM as prediction model

State equations:

\[
\begin{align*}
\rho_i(h + 1) &= \rho_i(h) + \frac{T}{L_i} \left[ \phi_i(h) + r_i(h) - \phi_{i+1}(h) - s_i(h) \right] \\
l_i(h + 1) &= l_i(h) + T \left[ d_i(h) - r_i(h) \right]
\end{align*}
\]

where \(s_i(h) = \frac{\beta_i}{1 - \beta_i} \phi_{i+1}(h)\).

Merge model:

\[
\begin{align*}
\text{IF} & \quad D_{i-1}(h) + d_i(h) + \frac{l_i(h)}{T} \leq S_i(h) \\
\text{THEN} & \quad \phi_i(h) = D_{i-1}(h), \quad r_i(h) = d_i(h) + \frac{l_i(h)}{T} \\
\text{ELSE} & \quad \phi_i(h) = \text{mid} \left\{ D_{i-1}(h), S_i(h) - d_i(h) - \frac{l_i(h)}{T}, (1 - p_i)S_i(h) \right\} \\
r_i(h) = \text{mid} \left\{ d_i(h) + \frac{l_i(h)}{T}, S_i(h) - D_{i-1}(h), p_iS_i(h) \right\}
\end{align*}
\]

where the demand and the supply of cell \(i\) [veh/h] are given by \(D_i(h) = \min \left\{ (1 - \beta_i)v_i \rho_i(h), F_i \right\}\) and \(S_i(h) = \min \left\{ w_i(\bar{\rho}_i - \rho_i(h)), F_i \right\}\).

Model parameters: split ratio \(\beta_i \in [0, 1]\), free flow speed \(v_i\) [km/h], congestion wave speed \(w_i\) [km/h], capacity \(F_i\) [veh/h], jam density \(\bar{\rho}_i\) [veh/km], on-ramp priority \(p_i \in [0, 1]\).
Some equations of the CTM are **nonlinear** (min, mid functions)

According to the framework of **Mixed Logical Dynamical** (MLD) systems, the nonlinear relations have been transformed into linear ones by introducing some sets of auxiliary variables (both binary and real) and some sets of equalities and inequalities

For instance

\[ D_i(h) = \min \{(1 - \beta_i)v_i \rho_i(h), F_i\} \quad i = 1, \ldots, N, \quad h = 0, \ldots, K - 1 \]

can be written as

\[ D_i(h) = (1 - \beta_i)v_i z_i^d(h) + (1 - \delta_i^d(h)) F_i \]

\[ (1 - \beta_i)v_i \rho_i(h) - F_i \leq D_i^{\max} (1 - \delta_i^d(h)) \]

\[ (1 - \beta_i)v_i \rho_i(h) - F_i \geq \epsilon + (D_i^{\min} - \epsilon) \delta_i^d(h) \]

\[ R_i^{\min} \delta_i^d(h) \leq z_i^d(h) \leq R_i^{\max} \delta_i^d(h) \]

\[ z_i^d(h) \geq \rho_i(h) - R_i^{\max} (1 - \delta_i^d(h)) \]

\[ z_i^d(h) \leq \rho_i(h) - R_i^{\min} (1 - \delta_i^d(h)) \quad i = 1, \ldots, N, \quad h = 0, \ldots, K - 1 \]

where \( \delta_i^d(h), \ z_i^d(h), \ i = 1, \ldots, N, \ h = 0, \ldots, K - 1, \) are respectively binary and real auxiliary variables

**The CTM in MLD form has a mixed-integer linear structure**
The finite horizon optimal control problem

Problem (Problem to be solved at time $k$ over a horizon of $K_p$ time steps)

Given:
- the initial conditions on the density and the queue length $\rho_i(k)$ and $l_i(k)$, $i = 1, \ldots, N$
- the demand of the cell before the first one $D_0(h), h = k, \ldots, k + K_p - 1$
- the supply of the cell after the last one $S_{N+1}(h), h = k, \ldots, k + K_p - 1$
- the on-ramp demands $d_i(h), i = 1, \ldots, N, h = k, \ldots, k + K_p - 1$

find the optimal control variables $r_i(h), i = 1, \ldots, N, h = k, \ldots, k + K_p - 1$, minimizing the adopted cost function $J(k)$ subject to the CTM in MLD form and all the constraints that characterize the physical problem.
Adopted control objectives $J(k)$

- Minimization of **TTS** $\rightarrow$ the FHOCP is mixed-integer linear

- Minimization of quadratic deviations of the state variables from given set-points $\rightarrow$ the FHOCP is mixed-integer quadratic

- Minimization of cases in which the state variables exceed given threshold values $\rightarrow$ the FHOCP is mixed-integer linear

- Minimization of the number of congested cells, properly weighted with the on-ramp queue lengths $\rightarrow$ the FHOCP is mixed-integer linear

\[
J(k) = \sum_{h=k}^{k+K_p-1} \sum_{i=1}^{N} L_i \rho_i(h) + l_i(h)
\]

\[
J(k) = \sum_{h=k}^{k+K_p-1} \sum_{i=1}^{N} \gamma_i^p (\rho_i(h) - \rho_i^*)^2 + \gamma_i^l (l_i(h) - l_i^*)^2
\]

\[
J(k) = \sum_{h=k}^{k+K_p-1} \sum_{i=1}^{N} \gamma_i^p \bar{\rho}_i(h) + \gamma_i^l l_i(h)
\]

\[
J(k) = \sum_{h=k}^{k+K_p-1} \sum_{i=1}^{N} \gamma_\delta (1 - \delta_i^m(h)) + \gamma_l l_i(h)
\]
The event-triggered MPC scheme

- In the classical MPC scheme a FHOCP is solved at each time step.
- In most cases this is redundant, especially if slight variations have happened in the system.
- We propose an event-triggered MPC scheme in which the control law is not updated at each time step but whenever a predefined set of conditions is verified.
- This set of conditions is named triggering rule.

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At $k = 0$ the FHOCP is solved determining $r^\circ_i(h|0)$, $i = 1, \ldots, N$, $h = 0, \ldots, K_p - 1$ and $r^\circ_i(0|0)$ is applied.

If at time step $k > 0$ the triggering rule is not met, the available control sequence $r^\circ_i(k|k_c)$ is applied, where $k_c$ is the time step in which the FHOCP has been solved for the last time (of course it must be $k - k_c < K_p$).

If at time step $k > 0$ the triggering rule is met, the FHOCP is solved determining $r^\circ_i(h|k)$, $i = 1, \ldots, N$, $h = k, \ldots, k + K_p - 1$ and $r^\circ_i(k|k)$ is applied.
The triggering rule

- at each time step $k$, the set of cells for which there is a relevant deviation of the real system behaviour from the predicted one is created according to the following logic

\[
\delta_{i}^{m}(k) \neq \hat{\delta}_{i}^{m}(k|k_c) \vee |\rho_{i}(k) - \hat{\rho}_{i}(k|k_c)| > \epsilon^{\rho} \vee |l_{i}(k) - \hat{l}_{i}(k|k_c)| > \epsilon^{l}
\]

then $i \in I(k)$

where $k_c$ is the previous triggering time step, $\epsilon^{\rho}$ and $\epsilon^{l}$ are threshold values

**Triggering rule to be verified at time $k > 0$**

\[
|I(k)| \geq \epsilon^{I} \vee k - k_c \geq K_p
\]

where $\epsilon^{I}$ is a given threshold
10 cells with 2 ramps, in the second and seventh cell

One hour of simulation, i.e. $K = 180$ ($T = 20$ [s])

Objective function: minimization of cases in which the state variables exceed given threshold values

Parameters of the event-triggered MPC approach: $\rho^t_i = 95$ [veh/km], $\forall i$, $\gamma_p = \gamma_l = 1$, $\epsilon^p = 15$ [veh/km], $\epsilon^l = 4$ [veh] and $\epsilon^l = 4$
In case the main aim of the traffic control scheme is to minimize the total time spent by the drivers, i.e. to maximize the total throughput, the best choice for the density threshold values is the critical density.

Performance improvement $\Delta J = 43\%$

TTS reduction $\Delta TTS = 7\%$

Computation ratio $\eta = 0.15$ (28 over 180)

Average time $\tau^{av} = 1.01 \text{ [s]}$

Maximum time $\tau^{max} = 1.61 \text{ [s]}$
Simulation results

- For some specific reasons (safety) and for a given time period, the objective can be to maintain a lower density

Performance improvement $\Delta J = 26\%$

TTS reduction $\Delta TTS = 2\%$

Computation ratio $\eta = 0.15$ (28 over 180)

Average time $\tau^{av} = 0.73$ [s]

Maximum time $\tau^{max} = 1.47$ [s]

The traffic density [veh/km] and the queue length [veh] (lower density threshold)
If the triggering rule is less strict (i.e. when $\epsilon^I$ increases), the index $\eta$ decreases, i.e. the number of computations of the FHOCP is strongly reduced.

The other indexes, regarding performances and computational times, does not change significantly.

<table>
<thead>
<tr>
<th>$\epsilon^I$</th>
<th>$\Delta J$</th>
<th>$\Delta TTS$</th>
<th>$\eta$</th>
<th>$\tau^{av}$</th>
<th>$\tau^{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.45</td>
<td>0.07</td>
<td>0.52</td>
<td>0.55</td>
<td>2.32</td>
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<tr>
<td>3</td>
<td>0.43</td>
<td>0.07</td>
<td>0.15</td>
<td>1.01</td>
<td>1.61</td>
</tr>
<tr>
<td>5</td>
<td>0.40</td>
<td>0.06</td>
<td>0.11</td>
<td>1.32</td>
<td>1.78</td>
</tr>
<tr>
<td>7</td>
<td>0.41</td>
<td>0.06</td>
<td>0.10</td>
<td>2.01</td>
<td>2.51</td>
</tr>
<tr>
<td>9</td>
<td>0.39</td>
<td>0.06</td>
<td>0.10</td>
<td>1.14</td>
<td>2.26</td>
</tr>
</tbody>
</table>

$\epsilon^I = \text{Triggering threshold}$
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The cluster-based distribution

- For large scale freeway systems we propose a cluster-based distributed control scheme.

- The freeway is divided into clusters of cells.
A cluster of cells is a subset of contiguous freeway cells which contains a single actuator, i.e. a single traffic light placed at the on-ramp.

The control variable associated with cluster $s$, $s = 1, \ldots, N_s$, is denoted with $u_s(k)$ and corresponds to the on-ramp traffic volume in the cluster.

The vector gathering all the state variables of cluster $s$, $s = 1, \ldots, N_s$, is denoted with $x_s(k)$, and includes the traffic densities of the cells of the cluster and the queue length of the on-ramp.

Let $N_s$ indicate the set of adjacent clusters of cluster $s$ (i.e. $N_s = \{s - 1, s + 1\}$).
According to the classification proposed by Scattolini (2009), the two proposed algorithms are partially connected and noniterative, i.e. each local controller exchanges some information with adjacent clusters only once within each sampling time.

The difference between Algorithm 1 (independent) and Algorithm 2 (cooperative) is in the cost function.

These distributed algorithms have been compared with a decentralized control approach, in which the control variables and the controlled ones are gathered into disjoint sets.

Algorithm 1 is a partially connected noniterative independent algorithm.

The local controller of cluster $s$ solves its own optimization problem, minimizing the local cost function $J_s(x_s(k), u_s(k))$ with respect to the sequence $u_s^o(k), \ldots, u_s^o(k + K_p - 1)$ related to cluster $s$ itself.

It uses the sequence $u_j^o(k), \ldots, u_j^o(k + K_p - 1)$, $j \in \mathcal{N}_s$, computed by the local controllers of the connected clusters, to determine the actual control sequence at time $k$:

$$u_s(k) = \nu_s u_s^o(k) + \sum_{j \in \mathcal{N}_s} \nu_j u_j^o(k)$$
Algorithm 2

- Algorithm 2 is a partially connected noniterative cooperative algorithm.
- The local controller of cluster $s$ solves its own optimization problem, minimizing the partial (nonlocal) cost function $J_{I_s}(k)$ with respect to the sequence related to cluster $s$ itself $u_{s_0}^o(k), \ldots, u_{s_0}^o(k + K_p - 1)$, and to the neighbors $u_{i,j}^o(k), \ldots, u_{i,j}^o(k + K_p - 1), j \in \mathcal{N}_s$.
- It uses the sequence $u_{s,j}^o(k), \ldots, u_{s,j}^o(k + K_p - 1), j \in \mathcal{N}_s$, computed by the local controllers of the connected clusters, to determine the actual control sequence at sampling time $k$:

$$u_s(k) = \nu_s u_{s_0}^o(k) + \sum_{j \in \mathcal{N}_s} \nu_j u_{i,j}^o(k)$$
In the decentralized control approach the control variables and the controlled ones are gathered into disjoint sets.

The local controller of cluster $s$ solves its own optimization problem, minimizing the local cost function $J_s(x_s(k), u_s(k))$ with respect to the sequence $u_s^o(k), \ldots, u_s^o(k + K_p - 1)$ related to cluster $s$ itself.

This sequence is the actual control sequence at time $k$. 

---

**The decentralized MPC scheme**

- **MPC**
  
  \[ J_s(x_s(k), u_s(k)) \]

- State $x_{s-1}(k)$
- State $x_s(k)$
- State $x_{s+1}(k)$

- $u_{s-1}(k)$
- $u_s(k)$
- $u_{s+1}(k)$

---

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Simulated scenario

- We have implemented the proposed control schemes with the C# programming language and we have adopted the MIQP solver Cplex 12.5 to solve each FHOCP.
- The simulation covers an horizon of $K = 150$ time steps (the sample time has been set as $T = 10$ [s]).
- 35 cells with 5 ramps (cells 3, 11, 18, 24, 30).
- 5 clusters: 1-6, 7-13, 14-20, 21-27, 28-35.

The traffic density [veh/km] in the open-loop case
Simulation results

- The behaviour of the **controlled system** is quite similar for the 4 considered cases (centralized, distributed 1, distributed 2, decentralized).

**The traffic density [veh/km] in the controlled case**

**The queue length [veh] in the controlled case**
## FHOCP characteristics

<table>
<thead>
<tr>
<th>Control scheme</th>
<th>Variables</th>
<th>Constraints</th>
<th>Average time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centralized</td>
<td>3636</td>
<td>8554</td>
<td>&gt;60</td>
</tr>
<tr>
<td>Decentralized-cluster 1</td>
<td>657</td>
<td>1556</td>
<td>0.14</td>
</tr>
<tr>
<td>Decentralized-cluster 2</td>
<td>756</td>
<td>1790</td>
<td>0.17</td>
</tr>
<tr>
<td>Decentralized-cluster 3</td>
<td>756</td>
<td>1790</td>
<td>0.21</td>
</tr>
<tr>
<td>Decentralized-cluster 4</td>
<td>756</td>
<td>1790</td>
<td>0.20</td>
</tr>
<tr>
<td>Decentralized-cluster 5</td>
<td>855</td>
<td>2024</td>
<td>0.55</td>
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<tr>
<td>Distributed 1-cluster 1</td>
<td>657</td>
<td>1556</td>
<td>0.14</td>
</tr>
<tr>
<td>Distributed 1-cluster 2</td>
<td>756</td>
<td>1790</td>
<td>0.17</td>
</tr>
<tr>
<td>Distributed 1-cluster 3</td>
<td>756</td>
<td>1790</td>
<td>0.21</td>
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<tr>
<td>Distributed 1-cluster 4</td>
<td>756</td>
<td>1790</td>
<td>0.20</td>
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<tr>
<td>Distributed 1-cluster 5</td>
<td>855</td>
<td>2024</td>
<td>0.55</td>
</tr>
<tr>
<td>Distributed 2-cluster 1</td>
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<tr>
<td>Distributed 2-cluster 5</td>
<td>1575</td>
<td>3715</td>
<td>1.52</td>
</tr>
</tbody>
</table>

All the experimental tests have been realized with a 2.2 GHz Intel(R) Core(TM) 2 Duo computer with 2 GB RAM.
Comparative analysis

Normalized values of cost reduction (improvement) with respect to the open-loop case, Total Time Spent reduction (improvement) with respect to the open-loop case and computational time.
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Conclusions

- We have proposed and tested different control algorithms with various control objectives oriented to be applied to the real show case.

- Future research will be devoted to couple the proposed algorithms with fast MPC techniques.

- Future research will be also aimed at designing distributed state estimation schemes (to keep the number of sensors on the freeway to the minimum).

- At present we are also investigating the possibility of integrating ramp metering with Variable Speed Limits control.


C. Canudas De Wit, L. Ramon Leon Ojeda, A.Y. Kibangou, “Graph constrained-CTM observer design for the Grenoble south ring”, *13th IFAC Symposium on Control in Transportation Systems*, 2012


Thanks for your attention!