Modeling, analysis and design of control tasks over wireless networking protocols

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Control of large-scale distributed and cooperating systems: Recent achievements within the Network of Excellence HYCON2
ECC14 Workshop #3, June 24, 2014
Challenges with Wired Control Networks

Wires are expensive
- Wires as well as installation costs
- Wire/connector wear and tear

Lack of flexibility
- Wires constrain sensor/actuator mobility
- Limited reconfiguration options

Restricted control architectures
- Centralized control paradigm
Paradigm shift towards wireless control architectures

“Removing cables undoubtedly saves cost, but often the real cost gains lie in the radically different design approach that wireless solutions permit. [...] In order to fully benefit from wireless technologies, a rethink of existing automation concepts and the complete design and functionality of an application is required.” Jan-Erik Frey, ABB
Applications of Wireless Control Networks

Industrial automation

Physical Security and Control

Supply Chain and Asset Management

Environmental Monitoring, Disaster Recovery and Preventive Conservation
Wireless Control Network

A collection of cooperating algorithms (controllers) designed to achieve a set of common goals, aided by interactions with the environment through distributed measurements (sensors) and actions (actuators) exchanged via a wireless communication network.
Opportunities vs challenges with Wireless Control Networks

Lower costs, easier installation
• Suitable for emerging markets

Broadens scope of sensing and control
• Easier to sense/monitor/actuate, opens new application domains

Compositionality
• Enables system evolution via through composable control systems

Runtime adaptation
• Control stability and performance are maintained in the presence of failures

Complexity
• Systems designers and programmers need suitable abstractions to hide the complexity from wireless devices and communication

Reliability
• Need for robust and predictable behavior despite wireless non-idealities

Security
• Wireless technology is vulnerable: security mechanisms for control loops
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Take into account communication protocol behavior!
ISO/OSI model for (wireless) communication protocols

- Open systems interconnection (OSI) model separates functional elements of a network into seven layers

![Diagram of ISO/OSI model with wireless link between Host A and Host B.]
ISO/OSI model for (wireless) communication protocols

- Open systems interconnection (OSI) model separates functional elements of a network into seven layers
- OSI model has allowed refinement of each layer independently

Host A
- Application
- Session
- Presentation
- Transport
- Network
- Data/Link
- Physical

Host B
- Application
- Session
- Presentation
- Transport
- Network
- Data/Link
- Physical

Wireless link

- Skype, youtube...
- TCP, UDP
- Routing strategy
- Scheduling, access to the wireless channel
- Coding, modulation, tx power
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ISO/OSI model for (wireless) communication protocols

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- Each layer only talks with the corresponding layer...

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Wireless link
ISO/OSI model for (wireless) communication protocols

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- OSI model has allowed refinement of each layer independently
- Each layer only talks with the corresponding layer...by exchanging packets with the layer below
Classical control loop

- Communication stack and medium is transparent to the control algorithm

\[ u(k) = f(y(k)) \]
Control loop over a wireless network

- Sensing and actuation data are relayed via the protocol stack layers

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Control loops over a wireless network

- Sensing and actuation data are relayed via the protocol stack layers
- Several feedback control mechanisms within separate communication layers

**Intra-layer control loops**

- Plant control law
- TCP congestion control
- Routing control
- Medium access control
- Power, coding & modulation control

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Control loops over a wireless network

- Sensing and actuation data are relayed via the protocol stack layers
- Several feedback control mechanisms within separate communication layers

Examples

- PID
- Tahoe, Reno, Cubic
- Routing Information Protocol (RIP), QoS
- Carrier Sense Multiple Access with Collision Detection (CSMA/CD)
- UMTS inner-loop power control

\[ \mathbf{u}(k) = f(\mathbf{y}(k)) \]
Control loops over a real wireless network
Control-aware networking and communication

Modify network protocols and radio links for better real-time control performance

\[ u(k) = f(y(k)) \]

Control loop

Wireless network

Sensing/actuation
Control-aware networking and communication

Modify network protocols and radio links for better real-time control performance

[Park et al 2011]
Network-aware control

Modify control algorithms to cope with communication imperfections
Network-aware control

Modify control algorithms to cope with communication imperfections
[Seiler&Sengupta 2001], [Jacobsson et al. 2004], [Sinopoli et al 2004], [Elia 2005],
[Imer et al 2006], [Braslavsky et al 2007], [Gupta et al 2007],
[Hespanha et al 2007], [Schenato et al 2007],
[Heemels et al 2011, 2012], [Chiuso et al 2014]
Co-design

Joint design of the control algorithm and the network protocol configuration

Control specification

A1  A2  S1

Wireless network
Co-design

Joint design of the control algorithm and the network protocol configuration

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Joint design of the control algorithm and the network protocol configuration

**Cross-layer interaction**: *Undesirable* properties of wireless link influence end-to-end network-control performance
Cross-layer adaptation & optimization

*Desirable* signalling between communication layers to improve overall performance
Cross-layer adaptation & optimization

Desirable signalling between communication layers to improve overall performance

Example: exploit plant and network feedback to decide actuation signal and power [D’Innocenzo et al 2012], [Gatsis et al 2013]

\[
\mathbf{u}(k) = f(\mathbf{y}(k))
\]
Cross-layer adaptation & optimization

Desirable signalling between communication layers to improve overall performance

Example: exploit plant and network feedback to decide actuation signal and coding [Tatikonda & Mitter 2004], [Nair et al 2007], [Quevedo et al 2010]
Cross-layer adaptation & optimization

Desirable signalling between communication layers to improve overall performance

Example: exploit plant and network feedback to decide actuation signal and access to channel

[Xu&Hespanha 2004], [Cogill et al 2007], [Li&Lemmon 2011], [Tabuada 2007], [Molin&Hirche 2009], [Rabi&Johansson 2009], [Anta&Tabuada 2010], [Donkers et al 2011]
Cross-layer adaptation & optimization

Desirable signalling between communication layers to improve overall performance

Example: exploit plant and network feedback to decide actuation signal and routing

[Mesquita et al 2012], [Jungers et al. 2014]
Co-design over time-triggered communication protocols

**Challenge:** Co-design the control algorithm and the communication protocol

[Diagram of a control system with layers labeled: Application, Session, Presentation, Transport, Network, Data/Link, Physical]

[Logo: WirelessHART, ISA100 Wireless]

Controller
Co-design over time-triggered communication protocols

*Challenge*: Co-design the control algorithm and the communication protocol (scheduling and routing)

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**Diagram Description**

The diagram illustrates a control system with the following components:

- **Application**
- **Session**
- **Presentation**
- **Transport**
- **Network**
- **Data/Link**
- **Physical**

The system includes a controller connected to various nodes labeled as **Vc** and **Vv**, with the output **y(kT)**. The system also features a ZOH (Zero-Order Hold) component, labeled as **P_T**. The diagram is associated with **Wireless HART** and **ISA100 Wireless** standards.

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**Diagram Components**

- **u(kT)**: Input signal
- **G_R**: Transfer function
- **P_T**: Control parameter
- **ZOH**: Zero-Order Hold
- **y(kT)**: Output signal
- **Vc** and **Vv**: Node labels
WirelessHART MAC (scheduling) and Network (routing) layers

- Time-triggered access to the channel
- Time divided in periodic frames
- Each frame divided in $\Pi$ time slots of duration $\Delta$
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Redundancy in data routing...

- ...enables detection and isolation of failures and malicious attacks
- ...makes system robust to short-term link failures (e.g. packet losses)
- ...makes system tolerant to long-term link failures
Close a control loop investigating two routing strategies:

1. **Single-path dynamic routing:** take into account switching behavior due to dynamic routing
2. **Multi-path static routing:** take into account algorithms to merge redundant data
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Wireless control networks as switching systems
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Different paths are associated with different delays.
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**Mathematical model:** \( x(t + 1) = Ax(t) + B(\sigma(t))v(t), \quad t \in \mathbb{N} \), where \( x(t) \) is the plant and network state, \( \sigma(t) \in \Sigma \) depends on routing/scheduling. The switching signal is considered as a disturbance.
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\[
A = \begin{pmatrix}
A_P & B_P & 0 & \cdots & 0 & 0 \\
0 & 0 & I & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & I & 0 \\
0 & 0 & 0 & \cdots & 0 & I \\
0 & 0 & 0 & \cdots & 0 & 0
\end{pmatrix}, \quad
B(\sigma(t)) = \begin{pmatrix}
B\delta_{\sigma(t),0} \\
I\delta_{\sigma(t),1} \\
\vdots \\
I\delta_{\sigma(t),D-2} \\
I\delta_{\sigma(t),D-1} \\
I\delta_{\sigma(t),D}
\end{pmatrix}
\]
Wireless control networks as switching systems

Different paths are associated with different delays.

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**Problem:** Design a controller $K(t)$ s.t. the closed loop system is asymptotically stable.

Given a state-feedback static controller $K(t)$, the closed loop systems is asymptotically stable iff the **Joint Spectral Radius** of $\{A + B(\sigma(t))K(t)\}_{\sigma(t)\in\Sigma}$ is smaller than 1.
Wireless control networks as switching systems

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**Insights:** Switching systems analysis and design is a crowded research area:

- Leverage special structure of matrices \( A \) and \( B(\sigma(t)) \) to provide tailored results that outperform classical results on general switching systems.
Wireless control networks as switching systems

Stabilizability depends on our knowledge of the (routing) switching signal $\sigma(t)$:
Wireless control networks as switching systems

Stabilizability depends on our knowledge of the (routing) switching signal $\sigma(t)$:

- No knowledge of routing signal, i.e. we cannot measure $\sigma(t)$: then $K(t) = K$, $\forall t \in \mathbb{N}$
  - Existence of non-linear controllers that outperform any linear controller
  - Approximation methods for the design of $K(t)$
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- **We can measure and keep memory of $\sigma(t)$**: then $K(t) = K(\sigma(t - d): \sigma(t))$
  - Pathologic switching sequences that invalidate stabilizability
  - Configure network nodes to avoid such sequences
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- **We can measure and keep memory of $\sigma(t)$ and have a finite horizon knowledge of future $N$ switching signals:** then $K(t) = K(\sigma(t - d): \sigma(t + N))$
  - If $N \geq \left( \frac{n + 2|D|}{2|D|} \right)$ stabilizability is guaranteed ($n$ plant state-space dimension, $|D|$ maximum delay)
  - Configure network nodes to design routing policies for appropriate time-windows
Wireless control networks as switching systems

- R.M. Jungers, A. D'Innocenzo, M.D. Di Benedetto. Modeling, analysis and design of linear systems with switching delays. Submitted for publication
- R.M. Jungers, A. D'Innocenzo, M. D. Di Benedetto. Further results on controllability of linear systems with switching delays. 9th IFAC World Congress, 2014
- R.M. Jungers, A. D'Innocenzo, M. D. Di Benedetto. How to control Linear Systems with switching delays. 13th ECC, 2014
Redundancy in data routing...

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- ...makes system robust to short-term link failures (e.g. packet losses)
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- Objective: stabilize the closed-loop system
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Fault tolerant control

\( F \) set of all configurations of links subject to a failure or a malicious intrusion

- Do not reconfigure the whole network (i.e. scheduling and routing) when a failure occurs: instead, only reconfigure neighbors of faulty nodes
- Do not add complexity to local communication to detect faulty or malicious nodes: instead, use plant dynamics and path redundancy
Fault tolerant control

Previous results on SISO LTI systems

- **Problem 1**: N&S Conditions on network to guarantee the existence of a stabilizing controller for the MCN dynamics $M_f$ associated to any $f \in F$
- **Problem 2**: N&S Conditions on network to design a dynamical system (FDI) able to detect and isolate any $f \in F$

Fault tolerant control

Extension to MIMO LTI systems

- A. D'Innocenzo, F. Smarra, M.D. Di Benedetto. Fault Tolerant Control of MIMO Multi-Hop Control Networks. Submitted for publication
- A. D'Innocenzo, M.D. Di Benedetto, F. Smarra. Fault detection and isolation of malicious nodes in MIMO Multi-hop Control Networks. 52nd IEEE CDC, 2013
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Robustness to packet losses via routing redundancy
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\[ u(k) = (\gamma_1 v(k - 1), \ldots, \gamma_D v(k - D))', \gamma = (\gamma_1, \ldots, \gamma_D) \]
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Closed-loop dynamics, no packet losses

\[ x(k + 1) = [A + B(\gamma)K(\gamma)]x(k) \]
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Closed-loop dynamics, packet losses

\[ x(k + 1) = [A + B(γ)K(γ) - B_{σ(k)}(γ)K(γ)]x(k) \]
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i.i.d. \( \{p_0, p_1, \ldots, p_N\} \)
Robustness to packet losses via routing redundancy

Theorem [Costa et al.]: the system is asymptotically mean square stable (MSS) if and only if 
\[ S = p_0 \bar{A}_0 + p_1 \bar{A}_1 + \ldots + p_N \bar{A}_N \]
is Schur stable: \( r(S) < 1 \).

\[ \bar{A}_i = (A + (B - B_i)K) \otimes (A + (B - B_i)K) \]
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Provide computationally efficient approximations leveraging the fact that packet loss probability is much smaller than correct tx probability.

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Gerschgorin disks (polynomial)
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- Gerschgorin disks (polynomial)
- Transmission mean square error minimization (convex)
Challenges in Wireless Control Networks

Modeling
• Formal interfaces between control algorithms and wireless communication protocols
• Compositional models for scalable analysis and design of multiple control loops

Analysis
• Quantify impact of wireless networking on control performance

Design
• Controller design incorporating wireless network properties
• Control-network co-design

Robustness
• Robust with respect to packet losses and delays
• Tolerant with respect to failures and malicious intrusions

Tools
• Formal verification and automatic (co-)design of networked systems
• Co-simulation of control algorithms, communication protocols and physical systems
The future steps of Wireless Control Networks...

Hubert Robert, *Scala di palazzo*
(Palazzo Farnese)
The future steps of Wireless Control Networks...

M.C. Escher, *Relativity* (1953 Lithograph)