A Distributed Control Strategy for Optimal Reactive Power Flow with power constraints

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Smart Microgrid

- High penetration of distributed energy resources
- High stochasticity in the production levels of energy
- Great potential performance improvement (*green inexpensive energy and a number of ancillary services*)
A **Smart Microgrid** is a portion of the electrical power distribution network

- that connects to the transmission grid (**utility**) at one point
- that is managed **autonomously** from the rest of the network
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to operate the Microgrid **optimally** while satisfying some **constraints** on how the microgrid interfaces with the rest of the network.
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Aim of the work

To develop a *distributed feedback* control strategy to minimize power distribution losses, by controlling the *reactive power* injected by the **Microgenerators** (ORPF problem)
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Reactive power: the intuition

Lossless electric components need current but they do not need electric power.

However, in order to bring this current to these components, some electric power is dissipated along the transmission line.
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Lossless electric components need current but they do not need electric power.

However, in order to bring this current to these components, some electric power is dissipated along the transmission line.

It is convenient that the current is provided by the closest generator, namely by the generator which is connected to the lossless component by a line with the smallest resistance.
A model of a microgrid

ELECTRICAL GRID MODELING
A model of a microgrid

Power lines: impedances i.e. linear constraints on currents and voltages

Microgenerators/loads: linear constraints on the (active and reactive) powers
A model of a microgrid

**Sinusoidal regime:** We assume that the system works at the sinusoidal regime at a certain fixed frequency.

\[ u(t) = |u| \cos(\omega t + \angle u) \]

**Phasorial notation:** The signal \( u(t) \) is described by a complex number \( u \in \mathbb{C} \)

\[ u = |u| e^{j\angle u} \]
A model of a microgrid

Variables of the model

- $u_k$ voltage at node k
- $i_k$ current at node k
- $j_e$ current at edge e
- $Z_e$ impedance at edge e
A model of a microgrid

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- $u_k$: voltage at node $k$
- $i_k$: current at node $k$
- $j_e$: current at edge $e$
- $Z_e$: impedance at edge $e$

Ohm’s law

$$u_k - u_h = Z_e j_e$$
A model of a microgrid

Utility as **ideal voltage** generator

The utility ensures that the voltage at the node 0 is equal to the nominal voltage $U_N$
A model of a microgrid

Utility as **ideal voltage** generator

The utility ensures that the voltage at the node 0 is equal to the nominal voltage $U_N$

$$u_0 = U_N \quad U_N \in \mathbb{R} \quad \text{fixed}$$
A model of a microgrid

The node $k$ injects or absorbs a complex power

$$u_k \bar{i}_k = s_k \quad s_k = p_k + j \ q_k$$

$$p_k = Re[s_k] \quad \text{active power}$$

$$q_k = Im[s_k] \quad \text{reactive power}$$
A model of a microgrid

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An intelligent node $k$ (generators) can

- Measure the voltage $u_k$
- Actuate the injected powers $p_k, q_k$
Grid equation

We stack the variables in the vectors \( p, q, u, i \)

The grid state is described by the static system of equations

\[
\begin{align*}
  u &= Yi \\
  u_0 &= U_N \\
  u_v i_v &= p_v + iq_v & v \neq 0
\end{align*}
\]

where \( Y \) is the admittance matrix of the grid.

There is a **non-linear relation** among voltages, currents and powers.
Grid equation

Key approximation (Bolognani and Zampieri TAC 13)

If $U_N$ is big enough, there exists a solution $u$ of the static system of equations

$$u = U_N \left( \mathbf{1} + \frac{1}{U_N^2} X \bar{S} + \frac{1}{U_N^4} \delta \right)$$

where $\mathbf{1}$ is the vector of all ones and $X$ is a complex matrix s.t.

$$\|\delta\| \leq 4\|X\|^2 \|s\|^2$$
Grid equation

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### Key approximation (Bolognani and Zampieri TAC 13)

If $U_N$ is big enough, there exists a solution $u$ of the static system of equations

$$u = U_N \left( 1 + \frac{1}{U^2_N} X \bar{S} + \frac{1}{U^4_N} \delta \right)$$

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Grid equation

We introduce the following block decomposition of the vectors, e.g., for the reactive power $q$

$$q = \begin{bmatrix} q_0 \\ q_G \\ q_L \end{bmatrix}$$

- $G$ is the *generators* set
- $L$ is the *loads* set
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Control objective

The cost to be minimized is the Power Losses (PL) on the grid lines

\[ \text{Power Losses} = \sum_e \text{Re}[Z_e]|j_e|^2 \]

\[ \min_{q_G} \sum_e \text{Re}[Z_e]|j_e|^2 \]

\[ \text{s.t.} \quad q^m \leq q_h \leq q^M \]

The constraints represent the microgenerators limited generation capabilities.
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**NON-CONVEX OPTIMIZATION PROBLEM**: difficult to solve in a distributed way.
Assumption

All power lines have the same inductance/resistance ratio

\[ Z(e) = e^{j\theta} R(e) \]

where \( R(e) \) is a real number and \( \theta \) is fixed.
Grid equation approximation

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q = \begin{bmatrix} q_0 \\ q_G \\ q_L \end{bmatrix} \rightarrow X = e^{i\theta} \begin{bmatrix} 0 & 0 & 0 \\ 0 & M & N \\ 0 & N^T & Q \end{bmatrix}
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Assumption

All power lines have the same inductance/resistance ratio

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where \( R(e) \) is a real number and \( \theta \) is fixed.

By adopting the same block decomposition as before, we have

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q = \begin{bmatrix} q_0 \\ q_G \\ q_L \end{bmatrix} \quad \rightarrow \quad X = e^{i\theta} \begin{bmatrix} 0 & 0 & 0 \\ 0 & M & N \\ 0 & N^T & Q \end{bmatrix}
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Key approximation

If $U_N$ is big enough, there exists a solution $u$ of the static system of equations

$$u = U_N \left( 1 + \frac{1}{U_N^2} X \bar{S} + \frac{1}{U_N^4} \delta \right)$$

Proposition

Consider the set of nonlinear equations. Node voltages satisfy

$$\begin{bmatrix} u_0 \\ u_G \\ u_L \end{bmatrix} = \begin{pmatrix} U_N 1 + \frac{e^{i\theta}}{U_N} \begin{bmatrix} 0 & 0 & 0 \\ 0 & M & N \\ 0 & N^T & Q \end{bmatrix} \begin{bmatrix} p_G - i q_G \\ p_L - i q_L \end{bmatrix} \end{pmatrix}$$
Control objective approximation

Exploiting the previous Proposition we can formulate the OPRF Problem as

\[
\min_{q_G} \sum_{e} \text{Re}[Z_e]|j_e|^2
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s. t. \( q_h^m \leq q_h \leq q_h^M, \quad h \in G \)
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\min_{q_G} \quad q_G^T \frac{M}{2} q_G + q_G^T N q_L
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\text{s.t.} & \quad q^m_h \leq q_h \leq q^M_h, \quad h
\end{align*}
\]
We have been inspired by the classical dual-ascent algorithm. The Lagrangian of the problem

\[ L(q_G, \lambda) = q_G^T \frac{M}{2} q_G^T + q_G^T N q_L + \lambda^T (q_G - q_G^M) \]

\( q_G \): the primal variable

\( \lambda \): the dual variable
We solve iteratively the OPRF problem proposing a dual-ascent like algorithm which guarantees that $q_h \leq q^M_h$, $h \in G$ in each iteration.
Dual-ascent algorithm

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computation of the minimizer

$$\bar{q}_G = \arg \min L(q_G, \lambda(t))$$
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1. Computation of the minimizer

$$\bar{q}_G = \arg \min L(q_G, \lambda(t))$$

2. Lagrange multipliers update

$$\lambda(t + 1) = \max\{\lambda(t) + \gamma(\bar{q}_G - q^M), 0\}$$
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3. Component-wise projection and actuation

\[
q_G(t + 1) = \text{proj}_{\{\infty, q_G^M\}}(\bar{q}_G)
\]
Convergence results

Convergence Results

From standard optimization results it can be shown that our feedback control strategy converges if

\[ \gamma \leq \frac{2}{\rho(M^{-1})} \]

where \( \rho(M^{-1}) \) is the \( M^{-1} \) spectral radius.
How to distribute?

CONTROLLER DISTRIBUTED IMPLEMENTATION
How to distribute?

When two generators are neighbors and can communicate with each other?
How to distribute?

When two generators are neighbors and can communicate with each other?

Generators $h$ and $k$ are said to be **neighbors** if the electric path connecting each other does not contain other generators.
How to distribute?

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$N_h$: set of neighbors of generator $h$
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Distributed Dual-ascent algorithm

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\bar{q}_G = \arg \min L(q_G, \lambda(t))
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Main issue is the computation of

$$\overline{q}_G = \arg \min L(q_G, \lambda(t)) = -M^{-1}Nq_L - M^{-1}\lambda(t)$$

where

- $M^{-1}$?
- loads are not monitored
- $N$ is unknown
Grid equation approximation

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$$M_{hk}^{-1} \neq 0 \iff h \text{ and } k \text{ are neighbors}$$

$M^{-1}$ is a sparse matrix and can be assumed locally known
Main issue is the computation of

\[ \bar{q}_G = \arg \min L(q_G, \lambda(t)) = -M^{-1}Nq_L - M^{-1}\lambda(t) \]

Exploiting the former **linear approximation** and **voltage measurements**, the agents can estimate \( \bar{q}_G \)

\[ \bar{q}_h \simeq q_h(t) + \sum_{k \in N_h} M^{-1}_{hk} \left( |u_h||u_k| \sin(\angle u_k - \angle u_h - \theta) - \lambda_k(t) \right) \]
\[ + \text{ infinitesimal terms on } U_N \]
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Agent \( h \) needs from neighbors \( \{u_k, \lambda_k\} \)
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Agent $h$ needs from neighbors $\{u_k, \lambda_k\}$
Controller Distributed Implementation

at each iteration, node $h$ gathers from its neighbors voltage measurements and Lagrange multipliers

$$\{u_k = |u_k| \exp(i \angle u_k), k \in N_h\}, \quad \{\lambda_k, k \in N_h\}$$
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computation of the minimizer

\[ \bar{q}_h \leftarrow q_h(t) + \sum_{k \in N_h} M_{hk}^{-1} (|u_h||u_k| \sin(\angle u_k - \angle u_h - \theta) - \lambda_k(t)) \]
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2. Lagrange multipliers updates

$$\lambda_h \leftarrow \max\{\lambda_h + \gamma(q_h - q_h^{\text{max}}), 0\}$$
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3. Component-wise projection and actuation

$$q_h \leftarrow \text{proj}_{[-\infty, q_{h,\text{max}}]}(\bar{q}_h)$$

Ruggero Carli

Optimal Reactive Power Flow
For our simulation we use a 4.8 kV testbed inspired from the standard test feeder IEEE37. Grey nodes are the agents.
Simulation results

![Graph showing Power Losses against Iteration]

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Optimal Reactive Power Flow
Simulation results

The graphs illustrate the simulation results for power loss and reactive power. The upper graph shows the power loss $J_{loss}$ in [W], with three lines representing different systems: uncontrolled, controlled, and optimum value. The lower graph displays the reactive power [VAR], with a line indicating the power injected by agent 4 and the maximum value.
Conclusions

Summary

1. feedback control strategy to minimize power losses controlling the amount of reactive power injected
2. alternating of measuring and actuating steps
3. convexified problem by linear approximation

On going work

1. robustness of the algorithm to noisy measurements
2. reactive power for voltage regulation
3. functional cost to incorporate active power
4. time-varying loads
Grid Model

- $u_v, i_v \in \mathbb{C}$ are node $v$ voltage and current injected phasorial notation (steady state)
- $p_v, q_v \in \mathbb{R}$ are the powers injected or absorbed by $v$
- We stack the variables in the vectors $p, q, u, i$
Toward Prosumers…

Consumer  →  Prosumer (Consumer + Producer)
Toward Prosumers...

Consumer \[\rightarrow\] Prosumer (Consumer + Producer)
Agents’ Assumptions

We assume the Microgenerators (Agents) have

- sensing capabilities (PMU)
- computational capabilities
- communication capabilities
- local knowledge of the grid topology
There exists a unique symmetric, positive semidefinite matrix $X \in \mathbb{R}^{n \times n}$ such that

$$
\begin{align*}
XY &= I - 1e_0 \\
Xe_0 &= 0
\end{align*}
$$

The matrix $X$ depends only on the topology of the grid and on the power lines impedance.
Territory Partitioning: Problem formulation

- consider an environment $Q$ convex
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- place $N$ robots at $p = \{p_1, ..., p_N\}$
- partition the environment into $v = \{v_1, ..., v_N\}$
- defined the expected wait time

$$H(v, p) = \int_{v_1} \|q - p_1\|^2 \varphi(q) dq + \ldots + \int_{v_N} \|q - p_N\|^2 \varphi(q) dq$$
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Multicenter function
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**GOAL:** Minimize $H(v, p)$ with the respect to $p$ and $v$
From optimality conditions to algorithms

**Theorem (Lloyd ’57 least-square quantization)**
- at fixed positions, optimal partition is Voronoi
- at fixed partition, optimal positions are centroids
- Lloyd algorithm: alternate p-v optimization

**Centroid of a given region**

\[
Cd(Q) = \left( \int_Q \varphi(q) \right)^{-1} \int_Q \varphi(q)
\]

Ruggero Carli

Minimalistic Coordination and Partitioning
Technical challenges

- **state space?** Is not finite-dimensional
  non-convex disconnected polygons
  arbitrary number of vertices

- **peer-to-peer** is not deterministic, ill-defined
  two regions could have the same centroid
  disconnected/connected discontinuity

- **Lyapunov function?**
  - **convergence theorems?**
  - **conditions on deterministic / random meetings?**